Heterogeneous Fixed Export Costs and the Division of Labor

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Abstract

We theoretically investigates that how firms decide to exports and the extent of the division of labor under heterogeneous fixed export costs. In the equilibrium, exporters and non-exporters coexists and all exporters behave as borderline firms. Exporters promote the division of labor more strongly than non-exporters. A decrease in trade costs raises the cut off export fixed costs. It expands firm size and promotes the division of labor of exporters, while it shrinks firm size and make non-exporters refrain from the division of labor. These links between the cut off fixed export costs and the division of labor of exporters and non-exporters bring a new insight for the research line of trade and heterogeneous fixed export costs

Keywords: heterogeneous fixed export costs; division of labor within firms; export decision

JEL classification numbers : F12

1 Introduction

How do firms decide to export ? These questions are important theoretically and empirically. To answer the questions, many trade economists have studied the trade model with firm heterogeneity since Melitz (2003). There are two problem in these models.

The first problem is about fixed export costs. In explaining this division, fixed export costs, such as distributing costs and advertising expenses, play a key role. In this regards, many trade models have assumed that fixed export costs are identical across firms. Is this

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assumption empirically valid ? Bugameli and Infante (2003) emphasized the importance of ability to collect the information of export market using a survey of Italian manufacturing firms. This implies that fixed export costs. are very different from each other.

The second problem is about firm organization. Many trade models have assumed that firm productivity is exogenous but many studies have indicated that trade liberalization reorganizes firm structure and changes firm productivity. In particular, Zadeh (2013) showed that trade liberalization changes the extent of the division of labor within firms. We focus on the division of labor within firms for firm organization.

It is natural to think that heterogeneous fixed export costs and the extent of the division of labor within firms affect export decision. However, there are quite few papers to analyze this relationship. Then, in this paper we make clear theoretically that how firms decide to exports and the extent of the division of labor under heterogeneous fixed export costs.

We adopt the same model as that of Shintaku (2015,a) for an autarkic economy. We incorporate heterogeneous fixed export costs following Jorgensen and Schroder (2008). That is, firms engage in investment to start a business. After that, firms can observe export fixed costs which are random variables. Then, firms decide to export and the extent of the division of labor. The model determines the firm size and the cutoff value of export fixed costs simultaneously. For firms which have such a cutoff value, to export or not are indifferent.

This paper's main results are as follows. In the equilibrium, exporters and non-exporters coexists and all exporters produce output and input labor by the same amount as borderline firms. Exporters promotes the division of labor stronger than non-exporters. A decrease in trade costs raises the cut off export fixed costs. It reduces the number of firms, and non-exporters, while raising the number of exporters. It affects not only output of exporters but also that of non-exporters. It expand firm size and promotes the division of labor of exporters, while it shrinks firm size and refrains the division of labor of non-exporters.

A few papers analyzes heterogeneous fixed export costs. Schhmitt and Yu (2001) indicated that a decrease in transport costs and an increase in fixed costs for domestic market raise the number of traded goods. These results mean a positive link between scale economies and the volume of intra-industry trade. Jorgensen and Schroder (2006) presented a model similar to Schhmitt and Yu (2001) but focused on the tariff reduction in trade liberalization. They indicated that the sum of available home and foreign varieties increases for small tariffs. Furthermore, welfare increases for small tariffs and falls for large tariffs. That is, there exists a welfare maximization tariff. These models impose zero profit condition for non-exporters, but Jorgensen and Schroder (2008) does not impose it. Jorgensen and Schroder (2008) rather treats entry process such as Melitz (2003). That is, firms must pay sunk cost to enter the market and after the entry, they observe their fixed export costs. In such a model, Jorgensen and Schroder (2008) indicated that there exists a welfare maximization tariff as Jorgensen, Philipp and Schroder (2006). This paper adopt entry process following Jorgensen and Schroder (2008). While the above models treats constant marginal cost model, however, this paper treats variable marginal cost model based on the division of labor. Then, two types of firms which have different extents of the division of labor are generated endogenously. Those extents depend on the cut off fixed export costs. These links bring a new insight for the research line of trade and heterogeneous fixed export costs.

The rest of this paper is organized as follows. Section 2 analyzes trading equilibrium. Section 3 analyzes trade liberalization. Finally, we present the conclusion and Appendix.

2 Trading equilibrium

We adopt the same model as that of Shintaku (2015,a) for an autarkic economy. We extend the model by incorporating heterogeneous fixed export costs following Jorgensen and Schroder (2008). There are two identical countries (home and foreign). We focus on home country without loss of generality. We use superscripts e and ne for variables of exporters and non-exporters, respectively. We focus on an equilibrium in which exporters and non-exporters coexist.

2.1 Entry and heterogeneous export fixed costs

Firms investment wf_e to start a business. Representative household finance wf_e. We let M be the number of firms which engage in the investment. We focus on an equilibrium in which all firms which started a business do not exit. That is, M is equal to the number of operating firms. After the investment, firms observe the degree of difficulty of accessing export market, $\alpha \in [0, \infty)$. The random variable, α has a probability density function, $g(\alpha)$ and cumulative distribution function, $G(\alpha)$. Firms decide to enter the export market. For the firms which have $\bar{\alpha}$, whether the firms should export or not is indifferent. We call such firms "borderline firms". $G(\bar{\alpha})M$ and $[1 - G(\bar{\alpha})]M$ of firms are exporters and non-exporters, respectively. After production and sale, all firms die with probability 1 following Jorgensen and Schroder (2008).

All firms must pay fixed costs FC_d to operate in the domestic market. FC_d is given by wf_d. Firms which have α must pay fixed cost FC_x to enter the export market. FC_x is given by $FC_x(\alpha) = \alpha f_x$. Therefore, total cost function of non-exporters and exporters are given sa follows:

$$TC^{ne}(y^{ne}) = VC(y^{ne}) + FC_d = (2\gamma f y^{ne})^{1/2} + wf_d,$$

$$TC^{e}(y^{e}_t, \alpha) = VC(y^{e}_t) + FC_d + FC_x(\alpha) = (2\gamma f y^{e}_t)^{1/2} + w(f_d + \alpha f_x),$$

where y^e_t represents total output of exporters. We should note that total cost function of exporters, $TC^e(y^e_t, \alpha)$ depends on α.

2.2 Equilibrium allocation

Pricing rule of non-exporters and exporters are given by $PP^{ne} : p^{ne}_d = \mu MC(y^{ne})$ and PP^e : $p^e_d = \mu MC(y^e)$, respectively. That is, we can obtain $p^{ne}/w = (B+1)(2\gamma f)^{1/2}(y^{ne})^{-1/2}$. and $p^e_d/w = (B+1)(2\gamma f)^{1/2}(y^e_t)^{-1/2}$ respectively, where $B \equiv \mu/2-1$. Final good market clearing condition for non-exporter and exporter of home country are given by $y^{ne} = c_{ne}$ and $y^e_t = y^e_d + y^e_x = c_e + \tau c'^*_e$, respectively, where c_{ne}represents consumption of the home household for home non-exporters and $c_e$represents that for home exporters and c'^*_e represents consumption of the foreign household for imported brands from home country. Asterisk (*) in superscript represents economic entities of foreign country and "′" in superscript represents imported brands.

Relative quantity of exporters to non-exporters can be obtained from final good market clearing conditions of exporter's good, and those of non-exporter's good. These conditions and optimal pricing conditions gives the following condition, $RGMC$[1]:

$$RGMC : \frac{y^e_t}{y^{ne}} = (1+\tau^{1-\sigma})^{\frac{2}{2-\sigma}}. \tag{1}$$

(1) immediately derives the following proposition.

Proposition 1. *All exporters behave in the same way when outputs of non-exporters are positive.*

By multiplying both sides of p^{ne}/w and p^e_d/w by y_t, we have revenues, $r^{ne} = p^{ne}y^{ne} = w(B+1)(2\gamma f y^{ne})^{1/2}$ and $r^e_t = p^e_d y^e_t = w(B+1)(2\gamma f y^e_t)^{1/2}$.

From r^{ne}, r^e, $TC^{ne}(y^{ne})$, $TC^e(y^e)$, and (1), we can obtain the following conditions:

$$\frac{\pi^{ne}(y^{ne})}{w} = A\,(y^{ne})^{1/2} - f_d, \tag{2}$$

1) $RGMC$ can derived in the similar manner with (4) of Shintaku (2015,b)

$$\frac{\pi^e(y^{ne},\alpha)}{w} = A\,(y_t^e)^{1/2} - (f_d+\alpha f_x) = (1+\tau^{1-\sigma})^{\frac{1}{2-\sigma}} A\,(y^{ne})^{1/2} - (f_d+\alpha f_x), \tag{3}$$

where A is defined as follows:

$$A \equiv B(2\gamma f)^{1/2}.$$

We should note that profit of exporters, $\pi^e(y^{ne})/w$ depends on α and y^{ne} from $TC^e(y_t^e,\alpha)$ and (1).

We let $\tilde{\pi}$ represents expected profit before firms start a business and from (2) and (3), this is given by

$$\tilde{\pi} \equiv [1-G(\bar{\alpha})]\pi^{ne}(y^{ne}) + \int_0^{\bar{\alpha}} \pi^e(y^{ne},\alpha)g(\alpha)d\alpha.$$

Free-entry condition is given by

$$FE : \tilde{\pi} = wf_e. \tag{4}$$

This equation characterizes the relation between $\bar{\alpha}$ and y^{ne}.

We let expected value of α conditional on $\alpha \le \bar{\alpha}$ be $E[\alpha|\alpha \le \bar{\alpha}]$. That is, this is given by

$$E[\alpha|\alpha \le \bar{\alpha}] \equiv \int_0^{\bar{\alpha}} \alpha g(\alpha)d\alpha.$$

By using (4) and $E[\alpha|\alpha \le \bar{\alpha}]$, we can obtain equilibrium output of non-exporters, $y^{ne}_{T|\bar{\alpha}}$ for given $\bar{\alpha}$ as follows:

$$y^{ne}_{T|\bar{\alpha}} = \left[\frac{f_e + f_d + f_x E[\alpha|\alpha \le \bar{\alpha}]}{A(HG(\bar{\alpha})+1)}\right]^2, \tag{5}$$

where H is defined as follows:

$$H \equiv (1+\tau^{1-\sigma})^{1/(2-\sigma)} - 1.$$

Then, we adopt the following assumption to obtain the internal solution.

Assumption 1. *We assume $B > 0$. That is, $2 < \mu$ and $1 < \sigma < 2$ hold.*

From $1 < \sigma < 2$ of Assumption 1 and $\tau > 1$, $H > 0$ holds. From $B > 0$ of Assumption 1, A is also positive from $A \equiv B(2\gamma f)^{1/2}$.

(1) and (5) derive the following proposition.

Proposition 2. *Under Assumption 1, for all $\bar{\alpha} > 0$,*

1. $(p_d^{ne}/w)_{T|\bar{\alpha}}$, $(p_d^e/w)_{T|\bar{\alpha}}$, $y^e_{t,T|\bar{\alpha}}$, $y^{ne}_{T|\bar{\alpha}}$, $t^e_{T|\bar{\alpha}}$, $t^{ne}_{T|\bar{\alpha}}$, $l^e_{t,T|\bar{\alpha}}$, *and* $l^{ne}_{T|\bar{\alpha}}$ *are positive.*
2. $(p_d^{ne}/w)_{T|\bar{\alpha}} > (p_d^e/w)_{T|\bar{\alpha}}$, $y^e_{t,T|\bar{\alpha}} > y^{ne}_{T|\bar{\alpha}}$, $t^e_{T|\bar{\alpha}} > t^{ne}_{T|\bar{\alpha}}$, *and* $l^e_{t,T|\bar{\alpha}} > l^{ne}_{T|\bar{\alpha}}$ *hold.*

$y^e_{t,T|\bar{\alpha}} > y^{ne}_{T|\bar{\alpha}}$ can be explained as follows. If MC is constant (no division of labor), (1) becomes $y^e_t/yne = 1 + \tau^{1-\sigma}$. That is, even if there are is division of labor, $y^e_{t,T|\bar{\alpha}} > y^{ne}_{T|\bar{\alpha}}$ must holds to satisfies final good market conditions. When there are is division of labor, inequality of outputs is expanded. Without the division of labor, both type of firms have the same price for domestic market, p_d. In (1), from $2/(2-\sigma) > 1$, $(1+\tau^{1-\sigma})[2/(2-\sigma)] > 1 + \tau^{1-\sigma}$ holds. This indicates that exporters promote the division of labor stronger than non-exporters and then, $p^e_d < p^{ne}_d$ holds. This expands the inequality of outputs.

By substituting $y^e_{t,T|\bar{\alpha}}$ of (5) for $\pi^{ne}(y^{ne})$ of (2) and $\pi^e(y^{ne}, \alpha)$ of (3), we can obtain equilibrium profit of non-exporters and exporters, $\pi^{ne}_{T|\bar{\alpha}}$ and $\pi^e_{T|\bar{\alpha}}$ respectively, for given $\bar{\alpha}$ as follows:

$$\frac{\pi^{ne}_{T|\bar{\alpha}}}{w} = \frac{f_e + f_x E[\alpha|\alpha \leq \bar{\alpha}] - HG(\bar{\alpha})f_d}{(HG(\bar{\alpha}) + 1)},$$

$$\frac{\pi^e_{T|\bar{\alpha}}(\alpha)}{w} = \frac{(H+1)f_e + H(1 - G(\bar{\alpha}))f_d + [(H+1)E[\alpha|\alpha \leq \bar{\alpha}] - \alpha(HG(\bar{\alpha}) + 1)]f_x}{(HG(\bar{\alpha}) + 1)]}.$$

$\bar{\alpha}$ is characterized by the following cut off condition (CO):

$$CO : \pi^e(\bar{\alpha}) = \pi^{ne}(\bar{\alpha}).$$

CO, $\pi^{ne}_{T|\bar{\alpha}}/w$, and $\pi^e_{T|\bar{\alpha}}(\alpha)/w$, give non-linear equation which characterize equilibrium value of $\bar{\alpha}$, $\bar{\alpha}_T$ as follows:

$$\bar{\alpha}_T = \frac{H}{f_x} \frac{f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T]}{(HG(\bar{\alpha}_T) + 1)}. \tag{6}$$

Then, $(p^{ne}_d/w)_T$, $(p^e_d/w)_T$, $y^e_{t,T}$, y^{ne}_T, t^e_T, t^{ne}_T, $l^e_{t,T}$, and l^{ne}_T can be characterized.

Labor market clearing condition is given by

$$L = \underbrace{Mf_e}_{\textbf{Investment}} + \underbrace{\overbrace{[1 - G(\bar{\alpha})]Ml^{ne}}^{\textbf{Non-exporters}} + \overbrace{G(\bar{\alpha})Ml^e_t}^{\textbf{Exporters}}}_{\textbf{Production}}.$$

By substituting $l^e_{t,T}$ and l^{ne}_T for this equation, we can obtain M_T as follows:

$$M_T = \frac{2B}{2B+1} \frac{L}{f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T]}. \tag{7}$$

Then, we can characterize the equilibrium completely. We assume the following condition to obtain the equilibrium in which exporters and non-exporters coexist.

Assumption 2. *We assume $\bar{\alpha}_T > (Hf_d)/f_x$.*

Proposition 3. *If and only if Assumption 1 and 2 hold, the equilibrium in exporters and non-exporters coexist is determined uniquely.*

Proof: See Appendix A.

Assumption 2 certifies $(\pi^{ne}/w)_T > 0$. Otherwise, non-exporters exit.

3 Trade Liberalization

We consider a decrease in τ as trade liberalization. We can obtain the following proposition.

Proposition 4. *Under Assumption 1 and 2, the following properties hold.*

1. A decrease in τ raises the cut off value of fixed export costs, $\bar{\alpha}_T$.

2. A decrease in τ reduces the number of firms, M_T and non-exporters, $[1-G(\bar{\alpha}_T)]M_T$ while raises the number of exporters, $G(\bar{\alpha}_T)M_T$.

3. A decrease in τ shrinks firm size and refrains the division of labor of non-exporters (reduces y_T^{ne} and t_T^{ne}), while expand firm size and promotes the division of labor of exporters (raises y_T^e and t_T^e).

Proof: See Appendix B.

Property 1 of Proposition 4 is natural. A decrease in τ raises marginal revenue of exporters and this makes some non-exporters enter the export market.

Next, we consider property 2 of Proposition 4. A decrease in τ raises the the cut off $\bar{\alpha}$, directly raises the number of exporters and reduces that of non-exporters (cut off effect). However, exporters input a lot of labor into production and headquarter division and they absorb a lot of labor from the non-export firms and starting firms. This effect reduces the number of all firms (entry effect). In export firms, cut off effect dominates entry effect while in non-export firms, cut off effect is dominated by entry effect. We should note that entry effect does not describe the exit process such as Melitz (2003) but that it describes the entry process. That is, the number of non-exporters decreases because entry decreases.

Finally, we consider property 3 of Proposition 4. Remember that $\bar{\alpha}_T$ and y_T^{ne} are determined by free entry and cut off conditions. If τ decreases keeping $\bar{\alpha}_T$, profit of exporters increases. This violates free entry condition and then, causes new new entry. This makes exporters reduce output and this reduce also output of non-exporters from (1). If a decrease in τ raises $\bar{\alpha}_T$ keeping $y_{t,T}^e$, this reduce profit of exporters while this does not change profit of non-exporters. This violates cutoff condition and makes exporters raises output. This leads to a increase in y_T^{ne} following (1). In export firms, the latter effect dominates the former effect while in non-exporters, the former effect is dominates by the latter effect. Hence, export firms promotes the division of labor while non-export firms refrain from that.

4 Conclusion

In this paper we have extended the model of Shintaku (2015,a) to a trade model with heterogeneous fixed export costs following Jorgensen and Schroder (2008). In the equilibrium, exporters and non-exporters coexists and all exporters behave as borderline firms. Exporters promote the division of labor more strongly than non-exporters. A decrease in trade costs raises the cut off export fixed costs. It reduces the number of firms and non-exporters, while it raises the number of exporters. It affects not only output of exporters but also that of non-exporters. It expands firm size and promotes the division of labor of exporters, while it shrinks firm size and make non-exporters refrain from the division of labor. These links between the cut off fixed export costs and the division of labor of exporters and non-exporters bring a new insight for the research line of trade and heterogeneous fixed export costs.

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5 Appendix

Appendix A: Proof of Proposition 3

If profit of non-exporters is positive and $\bar{\alpha}_T$ exists uniquely, the other endogenous variables also exist uniquely. Hence, we focus on profit of non-exporters and existence and uniqueness of $\bar{\alpha}_T$.

Positive profit of non-exporters

(6) is can be rewritten as

$$f_e + f_x E[\alpha|\alpha \leq \bar{\alpha}_T] = -f_d + \bar{\alpha}_T[G(\bar{\alpha}_T) + 1/H]f_x.$$

By using this, we can rewrite $\pi^{ne}_{T|\bar{\alpha}}/w$ of (4) as follows:

$$[HG(\bar{\alpha}_T)+1]\frac{\pi^{ne}_{T|\bar{\alpha}}}{w} = [HG(\bar{\alpha}_T)+1]\left(\frac{\bar{\alpha}_T f_x}{H} - f_d\right).$$

This implies that $\pi^{ne}_{T|\bar{\alpha}}/w > 0$ is equivalent to $\bar{\alpha}_T > (Hf_d)/f_x$. Q.E.D.

Existence and uniqueness of $\bar{\alpha}_T$

(6) is can be rewritten as

$$\underbrace{\bar{\alpha}_T[HG(\bar{\alpha}_T)+1]}_{K(\bar{\alpha}_T)} = \underbrace{\frac{H}{f_x}\left(f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T]\right)}_{J(\bar{\alpha}_T)}. \tag{A.1}$$

We let $K(\bar{\alpha})$ be $\bar{\alpha}_T[HG(\bar{\alpha})+1]$ and let $J(\bar{\alpha})$ be $(H/f_x)\left(f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}]\right)$. We should note that $K' > 0$, $J' > 0$, $J(0) = H(f_e + f_d)/f_x > 0 = K(0)$. If $K' > J'$ holds for all $\bar{\alpha} > 0$, $\bar{\alpha}_T$ exists uniquely from monotonicity of K and J. Such a situation can be explained by Figure 1.

We show $K' > J'$ holds for all $\bar{\alpha} > 0$ as follows:

$$\begin{aligned} K'(\bar{\alpha}) - J'(\bar{\alpha}) &= [(HG(\bar{\alpha})+1) + H\bar{\alpha}g(\bar{\alpha})] - \frac{H}{f_x} f_x \frac{dE[\alpha|\alpha \leq \bar{\alpha}]}{d\bar{\alpha}} \\ &= [(HG(\bar{\alpha})+1) + H\bar{\alpha}g(\bar{\alpha})] - H\bar{\alpha}g(\bar{\alpha}) \\ &= HG(\bar{\alpha}) + 1 > 0, \quad \text{for all} \quad \bar{\alpha}. \end{aligned}$$

Q.E.D.

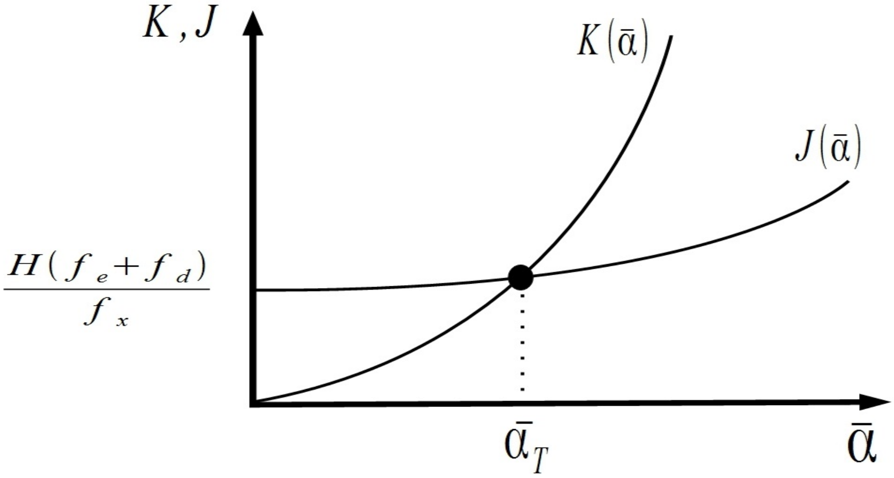

Figure 1: Relative final good market clearing and free-entry conditions.

Appendix B: Proof of Proposition 4

From $1 < \sigma < 2$ of Assumption 1, we can get the following condition:

$$\frac{dH}{d\tau} = \frac{1}{2-\sigma}(1+\tau^{1-\sigma})^{(\sigma-1)/(2-\sigma)} \underbrace{(1-\sigma)}_{-} \tau^{-\sigma} < 0.$$

Property 1

(A.1) can be rewritten as

$$\bar{\alpha}_T f_x \left[G(\bar{\alpha}_T) + \frac{1}{H}\right] = f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T].$$

By differentiating this equation with respect to τ, we can obtain the following equation:

$$\frac{d\bar{\alpha}_T}{d\tau} f_x \left[G(\bar{\alpha}_T) + \frac{1}{H}\right] + \bar{\alpha}_T f_x \left(g(\bar{\alpha}_T)\frac{d\bar{\alpha}_T}{d\tau} - \frac{1}{H^2}\frac{dH}{d\tau}\right) = f_x \frac{dE[\alpha|\alpha \leq \bar{\alpha}_T]}{d\bar{\alpha}_T}\frac{d\bar{\alpha}_T}{d\tau}$$

From $dE[\alpha|\alpha \leq \bar{\alpha}_T]/d\bar{\alpha}_T = \bar{\alpha}_T g(\bar{\alpha}_T)$, we can obtain the following equation:

$$\frac{d\bar{\alpha}_T}{d\tau}\left[f_x\left(G(\bar{\alpha}_T) + \frac{1}{H}\right)\right] + (\bar{\alpha}_T f_x)\frac{(-1)}{H^2}\frac{dH}{d\tau} = 0. \quad \text{(B.1)}$$

From $dH/d\tau < 0$, we can obtain $d\bar{\alpha}_T/d\tau < 0$. Q.E.D.

Property 2

By differentiating (7) with respect to $\bar{\alpha}_T$, we can obtain the following equations from $dE[\alpha|\alpha \leq \bar{\alpha}_T]/d\bar{\alpha}_T = \bar{\alpha}_T g(\bar{\alpha}_T)$:

$$\begin{aligned}\frac{dM_T}{d\bar{\alpha}_T} =& -\frac{2BL}{2B+1}\frac{dE[\alpha|\alpha \leq \bar{\alpha}_T]/d\bar{\alpha}_T f_x}{(f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T])^2} \\ =& -\frac{2BL}{2B+1}\frac{\bar{\alpha}_T g(\bar{\alpha}_T) f_x}{(f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T])^2} \\ =& -M_T\frac{\bar{\alpha}_T g(\bar{\alpha}_T) f_x}{f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T]} \\ <& \ 0.\end{aligned} \tag{B.2}$$

From $d\bar{\alpha}_T/d\tau < 0$, we can obtain

$$\frac{dM_T}{d\tau} = \underbrace{\frac{dM_T}{d\bar{\alpha}}}_{-}\underbrace{\frac{d\bar{\alpha}}{d\tau}}_{-} > 0.$$

From this equation and $d\bar{\alpha}_T/d\tau < 0$, we can obtain

$$\frac{d[1-G(\bar{\alpha}_T)]M_T}{d\tau} = \underbrace{-g(\bar{\alpha}_T)\frac{d\bar{\alpha}_T}{d\tau}M_T}_{\textbf{cut off effect (+)}} + \underbrace{[1-G(\bar{\alpha}_T)]\frac{dM_T}{d\tau}}_{\textbf{entry effect (+)}} > 0.$$

From this equation and $d\bar{\alpha}_T/d\tau < 0$, we can obtain

$$\begin{aligned}\frac{dG(\bar{\alpha}_T)M_T}{d\tau} &= \underbrace{g(\bar{\alpha}_T)\frac{d\bar{\alpha}_T}{d\tau}M_T}_{\textbf{cut off effect (−)}} + \underbrace{G(\bar{\alpha}_T)\frac{dM_T}{d\tau}}_{\textbf{entry effect (+)}} \\ &=\frac{\bar{\alpha}_T}{d\tau}\left(g(\bar{\alpha}_T)M_T + G(\bar{\alpha}_T)\frac{dM_T}{\bar{\alpha}_T}\right) \\ &=\frac{\bar{\alpha}_T}{d\tau}\left(g(\bar{\alpha}_T)M_T - G(\bar{\alpha}_T)M_T\frac{\bar{\alpha}_T g(\bar{\alpha}_T)f_x}{f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T]}\right) && \text{by (B.2)} \\ &=\frac{\bar{\alpha}_T}{d\tau}g(\bar{\alpha}_T)M_T\left(1 - G(\bar{\alpha}_T)\frac{\bar{\alpha}_T f_x}{f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T]}\right) \\ &=\frac{\bar{\alpha}_T}{d\tau}g(\bar{\alpha}_T)M_T\left(\frac{[f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T] - G(\bar{\alpha}_T)\bar{\alpha}_T f_x}{f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T]}\right) \\ &=\frac{\bar{\alpha}_T}{d\tau}g(\bar{\alpha}_T)M_T\left(\frac{[\bar{\alpha}_T f_x(G(\bar{\alpha}_T)+1/H)] - G(\bar{\alpha}_T)\bar{\alpha}_T f_x}{f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T]}\right) && \text{by (A.1)} \\ &= \underbrace{\frac{\bar{\alpha}_T}{d\tau}}_{-}\underbrace{g(\bar{\alpha}_T)M_T\left(\frac{(\bar{\alpha}_T f_x)/H}{f_e+f_d+f_xE[\alpha|\alpha \leq \bar{\alpha}_T]}\right)}_{+} < 0.\end{aligned}$$

Hence, this effect is negative. Q.E.D.

Property 3

By substituting $\bar{\alpha}_T$ for $y^{ne}_{T|\bar{\alpha}}$ of (3), we can obtain the following equations:

$$\begin{aligned} y^{ne}_T &= \left[\frac{f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T]}{A(HG(\bar{\alpha}_T)+1)}\right]^2 \\ &= \frac{1}{A^2}\left[\frac{f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T]}{(HG(\bar{\alpha}_T)+1)}\right]^2 \\ &= \frac{f_x^2}{A^2H^2}\left[\frac{H}{f_x}\frac{f_e + f_d + f_x E[\alpha|\alpha \leq \bar{\alpha}_T]}{(HG(\bar{\alpha}_T)+1)}\right]^2 \\ &= \left(\frac{f_x\bar{\alpha}_T}{AH}\right)^2. \end{aligned} \tag{B.3}$$

By differentiating (B.3) with respect to τ, we can obtain

$$\frac{dy^{ne}_T}{d\tau} = 2\left(\frac{f_x\bar{\alpha}_T}{AH}\right)\left(\frac{f_x}{A}\right)\frac{\overbrace{(d\bar{\alpha}_T/d\tau)}^{-}H - \bar{\alpha}_T\overbrace{(dH/d\tau)}^{-}}{H^2}.$$

Hence, $dy^{ne}_T/d\tau > 0(\leq 0)$ is equivalent to $(d\bar{\alpha}_T/d\tau)H > (\leq)\ \bar{\alpha}_T(dH/d\tau)$. This is equivalent to

$$\underbrace{-\frac{(dH/H)}{(d\tau/\tau)}}_{(+)} > (\leq)\ \underbrace{-\frac{(d\bar{\alpha}_T/\bar{\alpha}_T)}{(d\tau/\tau)}}_{(+)}. \tag{B.4}$$

That is, $dy^{ne}_T/d\tau$ depends on whether elasticity of $\bar{a}_T$ for τ is grater than that of H.

From (B.3) and (1), we can obtain

$$y^e_{t,T} = (H+1)^2 y^{ne}_T = \left(\frac{f_x\bar{\alpha}_T}{A}\frac{H+1}{H}\right)^2.$$

By differentiating this equation with respect to τ, we can obtain the following equation

$$\frac{dy^e_{t,T}}{d\tau} = 2\left(\frac{H+1}{H}\frac{f_x\bar{\alpha}_T}{A}\right)\left(\frac{f_x}{A}\right)\left[\underbrace{\frac{H+1}{H}\frac{d\bar{\alpha}_T}{d\tau}}_{-} + \underbrace{\frac{d[(H+1)/H]}{d\tau}\bar{\alpha}_T}_{+}\right],$$

where $d[(H+1)/H]/d\tau = -(dH/d\tau)/H^2 > 0$.

Hence, $dy^{e}_{t,T}/d\tau > 0$ $(\leq)$ is equivalent to

$$\underbrace{-\frac{(dH/H)}{(d\tau/\tau)}}_{(+)} > (\leq) \; \underbrace{-(H+1)\frac{(d\bar{\alpha}_T/\bar{\alpha}_T)}{(d\tau/\tau)}}_{(+)}. \tag{B.5}$$

Hence, (B.5) demands more stronger price effect to attain $dy^{e}_{t,T}/d\tau > 0$ than (B.4). That is, (B.5) more tends to attain $dy^{e}_{t,T}/d\tau < 0$ more than (B.4).

We analyze the relation in magnitudes between the elasticity of H and $\bar{a}_T$ for τ. We can rewrite (B.1) as follows

$$-\frac{d\bar{\alpha}_T}{d\tau}\frac{\tau}{\bar{\alpha}_T}\left(HG(\bar{\alpha}_T)+1\right) = -\frac{dH}{d\tau}\frac{\tau}{H} \tag{B.6}$$

(B.6) implies the elasticity of H for τ is grater than that of $\bar{a}_T$. From this result and (B.4), we can obtain $dy^{ne}_{T}/d\tau > 0$. This leads to $dt^{ne}_{T}/d\tau > 0$.

We can rewrite (B.6) as follows:

$$\begin{aligned}
-\frac{(dH/H)}{(d\tau/\tau)} - (-1)(H+1)\frac{(d\bar{\alpha}_T/\bar{\alpha}_T)}{(d\tau/\tau)} &= -\frac{(d\bar{\alpha}_T/\bar{\alpha}_T)}{(d\bar{\alpha}_T/d\tau)}\left(HG(\bar{\alpha}_T)+1\right) - (-1)(H+1)\frac{(d\bar{\alpha}_T/\bar{\alpha}_T)}{(d\tau/\tau)} \quad \text{by (B.6)} \\
&= -\frac{d(\bar{\alpha}_T/\bar{\alpha}_T)}{(d\bar{\alpha}_T/d\tau)}\left[\left[HG(\bar{\alpha}_T)+1\right] - (H+1)\right] \\
&= \underbrace{-\frac{d(\bar{\alpha}_T/\bar{\alpha}_T)}{(d\bar{\alpha}_T/d\tau)}}_{(+)} H \underbrace{\left[G(\bar{\alpha}_T)-1\right]}_{(-)} < 0.
\end{aligned}$$

Hence, these equations and (B.5) derive $dy^{e}_{t,T}/d\tau < 0$. This leads to $dt^{e}_{T}/d\tau > 0$. Q.E.D.